Rural-Urban Migration with Risk Aversion and Regional Uncertainty

Yarika Ruangsiri*
School of Economics, University of Nottingham

February 2004

Abstract

In this paper we analyse the role of risk on individuals' migration decisions in a modified Harris-Todaro model. The individuals are heterogeneous with respect to their preferences towards risk and regional uncertainties exist in both the manufacturing and agricultural sectors located in urban and rural areas respectively. The existence of rural risk has been recognised but has not been explicitly modelled in the Harris-Todaro framework. While the urban risk is a possibility of being unemployed, we assume that the rural risk is caused by random states of nature. The crucial point is that the rural risk affects agricultural production and therefore affects agricultural wage. Each individual compares his expected payoffs across sectors thus the migration decision is perfectly rational. The individuals' degrees of risk aversion and distribution of individuals' types within the population determine the migration equilibrium. Our analysis gives two interesting results which policy makers should consider in policy designing process. First, both risk averse and risk loving individuals may choose to stay in the same sector. The policy makers should take this possibility into account so that they can design a suitable policy or a set of policies that matches workers' needs and characteristics in each sector. This leads to our second finding: in a case where the risk averse individuals are in the urban areas, a policy of rural development that seems, at first sight, to be a desirable tool for reducing the level of urban unemployment may result in migration in both directions (into and out of the urban areas) creating high social and economic costs. Nevertheless our results are achieved under the assumption that we are able to rank individuals with respect to their preferences towards risk although location decisions may not be monotonic with respect to the degrees of risk aversion.

*This paper is part of my Ph.D. thesis written at the University of Nottingham. I would like to thank my supervisors, Prof. Richard Cornes and Dr. Claudio Zoli, for their valuable suggestions and comments during the writing of the thesis. Also I would like to thank Prof. Subrata Ghatak and Dr. Indraneel Dasgupta for their comments on this part of the thesis. I am responsible for all errors.

1 Introduction

In most developing countries, we often observe two related phenomena. First is the existence of wage differential between high-income (modern or manufacturing) sector located in urban areas and low-income (traditional or agricultural) sector located in rural areas. Second is the continuation of rural-urban migration despite the existence of urban unemployment.

The first leading theory of migration in a dual economy is the Lewis rural-urban migration model, Lewis (1954). The model describes the long-run migration process and the crucial assumption is an unlimited supply of labour which is available at subsistence agricultural wage. The Lewis model explains how an underdeveloped economy transforms its economic structures from a heavy emphasis on subsistence agriculture to a more modern (urbanised) economy. Nevertheless the model cannot explain the stated two phenomena.

The pioneering work of Harris and Todaro, Harris and Todaro (1970), captures such curious phenomena in a simple two-sector model. In the Harris-Todaro (H-T) model, the migration process is a response to urban-rural expected wage differences with the urban unemployment rate acting as an equilibrating force on such migration (Harris and Todaro, 1970). In equilibrium, the expected wages are equal across the two sectors. Over decades, the H-T model has been extended in several ways [see, for example, Ghatak, Levine and Wheatley Price (1996) and references cited therein]. Since the rural-urban migration decision involves risk of being unemployed in the urban areas, an interesting extension is to examine behaviour of individuals who are different with respect to their attitudes towards risk. Important works on migration that consider risk averse behaviour within families where a member of agricultural family migrates in order to diversify risk in family's total income are Stark (1980, 1984). Another cause of rural-urban migration of a risk averse agent is imperfection of money (capital) market in the rural areas, Katz and Stark (1986). These works imply that there exists also risk in the rural areas which involves uncertain rural income. Stark and Levhari (1982) suggest another cause of uncertain rural income which is risk in agricultural production itself.

Our concerns involve risks in both sectors and heterogeneous individuals with respect to their characteristics, such as their attitudes towards risk, thus it is natural to consider a migration model where there exists regional uncertainties across sectors. As far as I am concerned, the introduction of rural uncertainty has not been explicitly modelled in a H-T framework. What would happen to the migration pattern when the regional uncertainties are introduced? Under what circumstances will a risk averse migrate? Do the results lead to the same policy implication as proposed in existing H-T literature?

This paper addresses these questions. We assume that the rural uncertainty is caused by random states of nature which affect the level of total agricultural output and thus rural income. As mentioned earlier, the assumption has been suggested in Stark and Levhari (1982). Moreover a recent study on agricultural sector in Thailand, in which agricultural technology is primary and depends on weather condition, implies the rural uncertainty in agricultural output which can be mitigated by a crop diversification (a form of insurance against unpleasant states of nature), Coxhead and Plangpraphan (1998).

The paper is constructed as follows. In Section 2, we modify the H-T model to incorporate uncertainty in agricultural production assuming that the migration decision-maker is the individual. We formulate the model in terms of certainty equivalent which is derived via application of the expected utility theory. The advantage of doing so is that we can express the migration model in monetary unit which can be directly compared with the H-T model. Thus the motive of rural-urban migration is to maximise certainty equivalent. An individual's certainty equivalent in the urban areas is a product of the minimum wage and a transformed employment probability which depends on his degree of risk aversion. In the absence of rural uncertainty, his certainty equivalent in the rural areas is his real wage as in the H-T model.

In Section 3, we show that the migration equilibrium always exists and the migration pattern can be established. Given the values of uncertain parameters and endogenous variables of the model, the equilibrium depends on distribution of individuals' types across the whole population.

In Section 4, we use numerical examples to show some interesting results where (i) both risk loving and risk averse individuals migrate to the urban areas and (ii) a policy of rural development leads to migration in both directions. Thus our model offers another explanation of rural-urban migration by a risk averse agent. However the cause of rural uncertainty is not necessarily the random states of nature, it could be the imperfection of money market as suggested in Katz and Stark (1986). Moreover the possibility of a policy leading to migration in both directions is rather striking. The policy that seems to be desirable at first sight may not be so and some adjustments may be needed in order to prevent such possibility from occurrence. Nevertheless whether such possibility can actually occur in LDCs depend on further empirical research on variables and parameters associated with both the agricultural and manufacturing sectors. Finally we conclude our analysis in Section 5.

2 The Migration Model

The migration model is a modified H-T model where we introduce rural risk. The H-T model is presented in more details in Appendix A. Assume that the economy consists of two sectors: the agricultural sector locors: the agricultural sector locufacturing sector located in urban areas. The economy is a small open economy. Therefore prices of agricultural and manufacturing outputs are given by the world prices and can be normalised such that the agriculture and manufacture terms of trade is equal to 1. The labour market in each sector is perfectly competitive. Labour is homogeneous in terms of ability but is heterogeneous in terms of preference towards risk. The minimum wage $(\bar{w})$ is imposed in the manufacturing sector at the level above the market-clearing wage. The labour constraint, the manufacturing production function (Y_m) and the level of urban employment (n_m) are defined as in Appendix A equations (15), (18), and (20)

respectively:

$$n_{urb} + n_a = 1$$
$$Y_m = f_m(n_m) := F_m\left(n_m, \bar{K}_m\right)$$
$$n_m = f_m'^{-1}(\bar{w})$$

where n_{urb} and $\bar{K}_m$ are defined in Appendix A. Note that n_m is lower than the level of labour demand in the manufacturing sector at the market-clearing wage.

There exists wage differential, between the agricultural and manufacturing sectors, which induces migration. In most migration model, an individual chooses to work in the sector that gives him the highest wage (which is the manufacturing wage in our context). However this will not be the case in the migration model with labour distortion such as the H-T model in which the number of urban jobs is fixed given the level of the minimum wage. In each period, once all the urban jobs are randomly filled, all the excess workers in the urban areas become unemployed and earn zero income. Thus the risk of migrating to the urban areas is the chance of being unemployed. However if all the unemployed are absorbed in to the agricultural sector, the rural wage will be too low and there will always be incentive to migrate to the urban areas. Therefore individuals migrate to maximise their expected wages. If a migrant is unemployed, he may stay open-unemployed in the urban areas and wait for an available job in the next period or he may migrate back to the rural areas and accept low-wage agricultural job.

The expected urban wage is the product of the employment probability (p) and the minimum wage. The H-T model assumes that there is no risk in terms of employment probability in the agricultural sector, the expected rural wage is equal to the rural real wage (w_a), Appendix A equation (17) . The H-T migration equilibrium is reached when the expected wages in both sectors are equal: from Appendix A equation (21)

$$w_a = p\bar{w},\ p = \frac{n_m}{1 - n_a}$$

where n_a is the equilibrium number of rural labour force. In equilibrium, p is the same for each urban worker. Thus we can say that the urban uncertainty is described by a certain fixed income and an endogenous probability determined by the model.

In addition, we assume that there exists agricultural uncertainty in terms of production. This is due to the fact that the level of agricultural output depends also on random states of nature such as weather. Under a good condition, the level of output is higher than the quantity produced under a bad condition given the same labour input. The probability of having a good condition is given by a fixed probability π which is known to individuals. Hence the expected rural wage depends on π and n_a. The agricultural production function is no longer represented by the equation (16) in Appendix A.

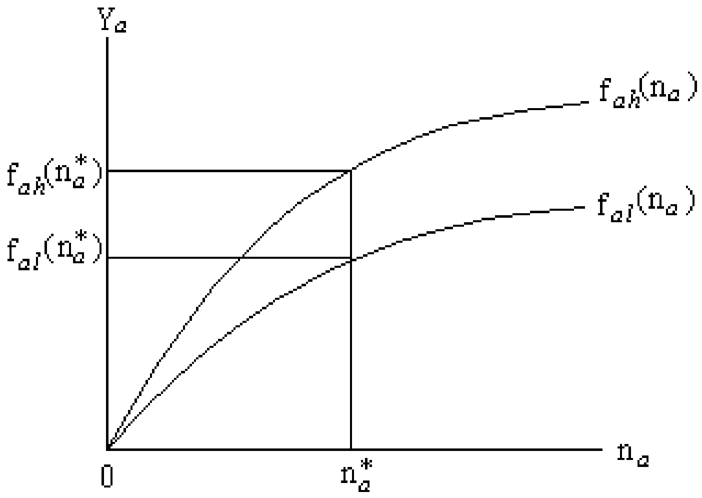

Figure 1: A stochastic production function in the agricultural sector.

Suppose that the agricultural production is represented by a stochastic-production function which is known to the individuals. That is the agricultural output is random and is determined by

$$Y_{a\theta} = f_{a\theta}(n_a) := F_{a\theta}(n_a, \bar{L}, \bar{K}), f'_{a\theta} > 0, f''_{a\theta} < 0 \tag{1}$$

where $Y_{a\theta}$ is total agricultural output that can be produced given n_a and θ which represents the randomly occurring events, $\theta \in \{l, h\}$, and $f'_{a\theta} = \frac{df_{a\theta}}{dn_a}$, $f''_{a\theta} = \frac{df'_{a\theta}}{dn_a}$.

Assume also that when $\theta = h$ which occurs with probability π, ex post or realised output is higher than when $\theta = l$ which occurs with probability $(1 - \pi)$, see Figure 1. Hence all the resolution of uncertainty does is to increase or decrease the productivity of a given bundle of nonstochastic inputs, $\bar{L}, \bar{K}$ and n_a given that the technology is unchanged.

Since firms are perfectly competitive and the minimum wage is imposed only in the urban areas, there is always full employment in the rural areas. The ex post rural wage is the ex post agricultural real wage which is defined as

$$w_a = \begin{cases} w_{al} = f'_{al}(n_a) \text{ if } \theta = l, \\ w_{ah} = f'_{ah}(n_a) \text{ if } \theta = h \end{cases} \tag{2}$$

where $w_{al} \leq w_{ah}$ for all values of n_a. Thus the rural uncertainty is described by a fixed probability and endogenous outcomes determined by the model. The expected rural wage is $\pi w_{ah} + (1 - \pi) w_{al}$.

If the individuals are risk neutral and are expected wage maximisers as assumed in the H-T model, the migration equilibrium will be reached when the expected wages in both sectors are equal. Nevertheless we assume that individuals are different with respect to their attitudes towards risk. Thus the value of a gamble (which is a migration decision in our context) to an individual is not, in general, equal to its expected monetary value, as stated by Bernoulli (1738): "rather, individuals place subjective value, or utilities, on monetary outcomes and the value of a gamble is the expectation of these utilities". Therefore in stead of assuming that individuals are expected wage maximisers, we assign a utility function on wages and assume that the individuals are expected utility maximisers.

In Section 2.1, we discuss the individual's utility function and show that the migration equilibrium is defined by a marginal individual whose expected utilities in both sectors are equal. This is done via the use of the expected utility theory which we summarise in Appendix B. Then we discuss, in Section 2.2, what we mean by an individual is *more risk averse than* another individual. In Section 2.3, we derive an alternative expression of the migration equilibrium in terms of certainty equivalent.

2.1 Utility Function

Since individuals are heterogeneous with respect to preferences towards risk, let N denote the total number of individuals which can be normalised to 1. Assume further that the individual i's utility function, $i = 1, ..., N$, is defined as

$$v^i(x) = \frac{x^{1-\gamma_i}}{1-\gamma_i} \tag{3}$$

where γ_i is the individual i's coefficient of relative risk aversion and x is the monetary outcome. Assume that $v^i(\cdot)$ is monotonically increasing and twice differentiable, its first and second derivatives with respect to x are

$$v^{i\prime}(x) = x^{-\gamma_i} \text{ and } v^{i\prime\prime}(x) = -\gamma_i x^{-\gamma_i - 1}.$$

Thus

$$r_A^i = \frac{\gamma_i}{x} \text{ and } r_R^i = \gamma_i$$

where r_A and r_R are the Arrow-Pratt coefficients of absolute and relative risk aversion at x, see Definition 6 in Appendix B. If $\gamma_i = 0$, the individual i is risk neutral. If $\gamma_i > 0$, he is risk averse whereas if $\gamma_i < 0$, he is risk loving.

The utility function defined in (3) exhibits both decreasing absolute risk aversion ($DARA$), i.e. $\frac{dr_A}{dx} < 0$, and constant relative risk aversion ($CRRA$), i.e. $\frac{dr_R}{dx} = 0$. An individual, whose $v(\cdot)$ exhibits $DARA$, is willing to take risk as he becomes wealthier. While an individual, whose $v(\cdot)$ exhibits $CRRA$, displays the same degree of risk aversion with respect to gambles that are proportional to his wealth as his wealth increases and is a stronger assumption than the latter.

Is the utility function exhibiting $DARA$ and $CRRA$ a reasonable functional form for our migration model? In my opinion, it seems so. The wealthier is more willing to take risk of being unemployed in the urban areas since he can afford to and that he is more likely to stay there to search for a job in the next period. On the other hand, the poorer individual does not have such financial support and therefore cannot afford to be unemployed. This behaviour is also supported by Xu (1992) which studies migration in China. Moreover the functional form is tractable.

Given that $v^i(\cdot)$ is defined as in (3). Applying the expected utility theorem, from Appendix B equations (28) and (29), the individual i's expected utility is

$$V^i(w) = \begin{cases} (1-\pi)\frac{(w^*_{al})^{1-\gamma_i}}{1-\gamma_i} + \pi\frac{(w^*_{ah})^{1-\gamma_i}}{1-\gamma_i} & \text{in rural areas,} \\ \left(\frac{n_m}{1-n^*}\right)\frac{\bar{w}^{1-\gamma_i}}{1-\gamma_i} & \text{in urban areas} \end{cases} \quad (4)$$

where $w^*_{a\theta} = f'_{a\theta}(n^*), \theta = l, h$, and n^* denotes the equilibrium level of rural labour force which is defined when

$$(1-\pi)\frac{(w^*_{al})^{1-\gamma^*}}{1-\gamma^*} + \pi\frac{(w^*_{ah})^{1-\gamma^*}}{1-\gamma^*} = \left(\frac{n_m}{1-n^*}\right)\frac{\bar{w}^{1-\gamma^*}}{1-\gamma^*} \quad (5)$$

where γ^* is the coefficient of relative risk aversion of the individual at the margin who is indifferent between staying in either regions.

The crucial assumption that allows us to establish migration pattern is that we can rank the individuals with respect to their preferences towards risk. There are several approaches that allow us to compare risk aversion across individuals using Definitions 4-6. An obvious way is to compare the degree of concavity of the individuals' utility functions. The individual h is more risk averse than the individual j if $v^h(\cdot)$ is *more concave* than $v^j(\cdot)$. This is equivalent to saying that the individual h's coefficient of absolute risk aversion is no less than the individual j's coefficient of absolute risk aversion for every x:

$$r^h_A(x) \geq r^j_A(x) \text{ for every } x. \quad (6)$$

Given the utility function defined in (3), the condition (6) is equivalent to

$$\gamma_h \geq \gamma_j \text{ for every } x. \quad (7)$$

However we use the concept of certainty equivalent to compare risk aversion across individuals. The certainty equivalent is the amount of money for which the individual is indifferent between the gamble $G(\cdot)$, which is a cumulative distribution function of the lottery over the set of all possible outcomes, and the certain amount of money (e) defined in Appendix B Definition 5. Therefore the expected utility in the urban areas written in terms of certainty equivalent can be linked directly to the H-T expected urban wage. This is shown in the following subsection.

2.2 Comparisons Across Individuals Using Certainty Equivalent

Given two utility functions $v^h(\cdot)$ and $v^j(\cdot)$, the condition for the statement "*the individual h is more risk averse than the individual j*" to hold can be expressed

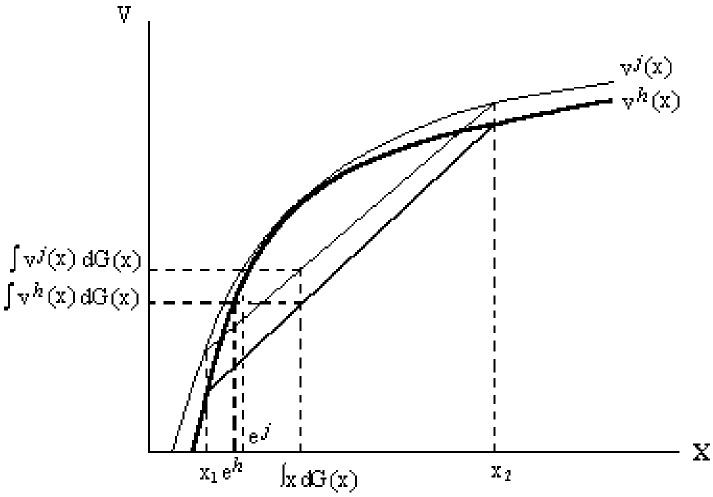

Figure 2: Utility functions of two risk averse individuals h and j denoted by $v^h(x)$ and $v^j(x)$ respectively: the individual h is more risk averse than the individual j.

in terms of certainty equivalent: that is

$$e^h \leq e^j \text{ for any } G(\cdot). \tag{8}$$

The condition can be shown using the graphical illustration. Figure 2 shows the case where the individual h is more risk averse than the individual j. Thus $v^h(\cdot)$ is more concave than $v^j(\cdot)$.

Given a lottery G that offers either x_1 or x_2 with probabilities p_1 and p_2 respectively where $p_1 + p_2 = 1$. The expected value of the possible outcomes is $\int xdG(x)$. By Appendix B Definition 5, it is clear from Figure 2 that the more risk averse individual requires a smaller amount of money which yields the same utility as the expected value of the lottery, $\int xdG(x)$: that is $e^h \leq e^j$, equation (8).

Given the definition of the certainty equivalent, Appendix B equation (32), and the expression of the individual i's expected utility, equation (4), the individual i's certainty equivalent is

$$e^i = \begin{cases} e_a^i & \text{in rural areas,} \\ e_m^i & \text{in urban areas} \end{cases} \tag{9}$$

where

$$\frac{\left(e_a^i\right)^{1-\gamma_i}}{1-\gamma_i} = (1-\pi)\frac{\left(w_{al}^*\right)^{1-\gamma_i}}{1-\gamma_i} + \pi\frac{\left(w_{ah}^*\right)^{1-\gamma_i}}{1-\gamma_i}$$

$$e_a^i = \left[(1-\pi)\left(w_{al}^*\right)^{1-\gamma_i} + \pi\left(w_{ah}^*\right)^{1-\gamma_i}\right]^{\frac{1}{1-\gamma_i}}$$

and

$$\frac{\left(e_m^i\right)^{1-\gamma_i}}{1-\gamma_i} = \left(\frac{n_m}{1-n^*}\right)\frac{\bar{w}^{1-\gamma_i}}{1-\gamma_i}$$

$$e_m^i = \left(\frac{n_m}{1-n^*}\right)^{\frac{1}{1-\gamma_i}}\bar{w}.$$

Notice that e_m^i is the product of the minimum wage and the transformed employment probability, $\left(\frac{n_m}{1-n^*}\right)^{\frac{1}{1-\gamma_i}}$.

Since we can rank individuals with respect to their degrees of risk aversion, for all $i = 1, ..., N$,

$$\gamma_1 \geq \gamma_2 \geq ... \geq \gamma_N$$

we can also rank them with respect to their certainty equivalent:

$$e_k^1 \leq e_k^2 \leq ... \leq e_k^N, k = a, m.$$

In both sectors, the certainty equivalent of the relatively more risk averse individual lies below that of the relatively less risk averse individual.

The individuals are expected utility maximisers, thus each individual migrates to maximise his certainty equivalent. Note that the certainty equivalent has monetary unit, so that the migration problem is similar to the H-T model where each individual migrates to maximise his expected wage. When $\gamma_i = 0$, the individual i is risk neutral and e_k^i is the sector k's expected wage, $k = a, m$. In addition to $\gamma_i = 0$, if there is no rural uncertainty that is either π or $(1-\pi)$ becomes 0, we have the H-T migration model.

Given (9), the migration equilibrium, equation (5), can be written as

$$e_a^* = e_m^*$$

$$\left[(1-\pi)\left(w_{al}^*\right)^{1-\gamma^*} + \pi\left(w_{ah}^*\right)^{1-\gamma^*}\right]^{\frac{1}{1-\gamma^*}} = \left(\frac{n_m}{1-n^*}\right)^{\frac{1}{1-\gamma^*}}\bar{w} \qquad (10)$$

where γ^* is the coefficient of relative risk aversion of the individual at the margin who is indifferent between staying in the rural areas or migrating to the urban areas.

The comparisons of risk aversion across individuals can be carried out in a similar fashion using the concepts of the certainty equivalent and the one related to the probability premium, in some non-expected utility theories in which the independence axiom of the expected utility theory does not hold, see Becker (2001).

3 Migration Decision

From (10), we know that, in equilibrium, the individual whose $\gamma = \gamma^*$ is indifferent between staying in either regions. However, there are more than one type of individuals and it is unclear whether those who have different degrees of risk aversion prefer to stay in the urban or rural areas. Fortunately, we can use geometric exposition similar to the one representing the H-T model, see Appendix A Figure 6, to illustrate the migration equilibrium and labour allocation. By equilibrium labour allocation, we mean proportions and types of labour across sectors in equilibrium. We find that the distribution of individuals' types across the whole population plays an important role in determining the equilibrium.

To work out the equilibrium labour allocation, we start by considering what the equilibrium level of labour in each sector would have been if the economy consisted of one type of individuals. This will allow us to compare individuals' preferences towards different areas. Given proportions of each type, we will be able to find the equilibrium level of rural labour force, denoted by n^*, and the individuals' types in each sector.

We start with the simplest case where the population is divided into two types: risk neutral and risk averse.

3.1 Two-type individuals

Consider the economy consisting of two types of individuals: the risk averse (type 1) whose γ is $\gamma_1 > 0$ and the risk neutral (type 2) whose γ is $\gamma_2 = 0$. Also let λ_i denote the proportion of type $i, i = 1, 2$, and $\lambda_1 + \lambda_2 = 1$.

Let n_i be the equilibrium level of rural labour force when the economy consists of type i only, $i = 1, 2$. From (9) and (10), n_i is defined when

$$\left[\pi\left(w_{ah}^i\right)^{1-\gamma_i} + (1-\pi)\left(w_{al}^i\right)^{1-\gamma_i}\right]^{\frac{1}{1-\gamma_i}} = \left(\frac{n_m}{1-n_i}\right)^{\frac{1}{1-\gamma_i}} \bar{w} \tag{11}$$

where $w_{a\theta}^i = f_{a\theta}'(n_i), \theta = l, h$.

The certainty equivalence approach allows us to draw the graphical exposition of the migration equilibrium, Figure 3, similar to that of the H-T model. The agricultural sector is drawn with origin 0_a while the manufacturing sector is drawn with origin 0_m. The length $0_a 0_m$ is equal to the total labour force, N, normalised to 1. In each sector, the certainty equivalent of the more risk averse individual lies below that of the less risk averse individual. The $R_i R_i$ and MHH_i respectively represent e_a^i and e_m^i, defined in equation (9).

Since type 2 are risk neutral, MHH_2 coincides with the expected urban wage curve in the H-T model. However the HH_1 part of the MHH_1 curve is not the rectangular hyperbola but is a transformed expected urban wage. The position of n_i is determined where the $R_i R_i$ and MHH_i curves intersect which occurs at Z_i.

From Figure 3, n_2 is on the left of n_1. Starting from the rural areas, the origin 0_a, over the range $n_a \in [0, n_2]$ both types prefer the rural areas. This is

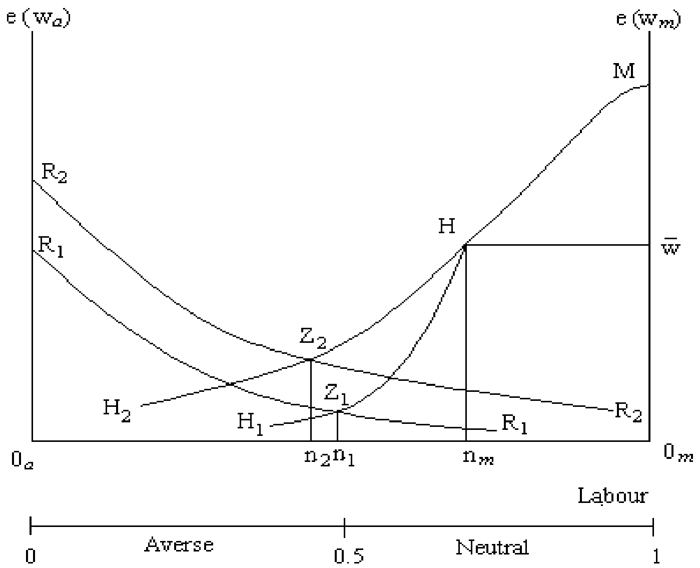

Figure 3: Geometric exposition of migration equilibrium in the two-type individual model, expressed in terms of certainty equivalent.

because, over such range, certainty equivalent of each type in the rural areas is higher than their corresponding certainty equivalent in the urban areas. For $n_a \in (n_2, n_1]$, only type 1 prefer the rural areas while type 2 strictly prefer the urban areas. For $n_a \in (n_1, 1]$, both types prefer the urban areas. Suppose that everyone is in the rural areas at the beginning of the period. Since there are too many people there and the rural wage is too low, type 2 will be the first group of people who migrate to the urban areas. Within the same sector, the certainty equivalent of both types never cross. Therefore members of each type tend to stick together and prefer the same areas unless they are indifferent between staying in either the rural or urban areas.

Figure 3 shows the case where the order of n_i is $n_2 < n_1$. That is we tend to find type 1 in the rural areas and type 2 in the urban areas: type 2 relatively prefer the urban areas. In equilibrium n^* will depend on the proportions of both types. To work out the equilibrium labour allocation, we compare the proportion of type i, λ_i, with the position of n_i starting from 0_a. We illustrate all the possible equilibrium allocations as follows:

Consider the case where $\lambda_1 \leq n_2$, all type 1 will strictly prefer the rural areas. Thus n^* is equal to n_2 and type 2 will be indifferent between staying in either regions. If the strict inequality holds, we also find a proportion of type 2 equal to $(n_2 - \lambda_1)$ in the rural areas.

If $n_2 < \lambda_1 < n_1$, all type 1 will strictly prefer the rural areas while all type 2 will strictly prefer the urban areas. In equilibrium, n^* is exactly equal to λ_1.

If $n_1 \leq \lambda_1$, n^* will be equal to n_1: type 2 strictly prefer the urban areas while type 1 are indifferent between staying in either regions. If the strict inequality holds, a portion of type 1 equal to $(\lambda_1 - n_1)$ will be in the urban areas.

From Figure 3, suppose that $\lambda_1 = \lambda_2 = \frac{1}{2}$, n^* is equal to n_1. Since n_1 is slightly less than λ_1, all the risk neutral and a few risk averse individuals will be in the urban areas while the rest of the risk averse will be in the rural areas.

On the other hand, if the graph shows the case where $n_1 < n_2$, the migration decisions of both types will be in the opposite directions of what has just been described above. In equilibrium, we are more likely to find type 1 in the urban areas and type 2 in the rural areas. The risk associated with the manufacturing sector is relatively lower than the rural risk. We can also increase the number of types as long as it is finite. The migration decision follows the same logic.

3.2 Three-type individuals

Let the economy consist of three types of individuals. In addition to the previous two-type case, we add the risk loving (type 3) whose γ is $\gamma_3 < 0$. Also λ_i represents the proportion of the type i individuals, $i = 1, 2, 3$, where $\lambda_1 + \lambda_2 + \lambda_3 = 1$.

The certainty equivalent of each type in each sector can be drawn in the same space as done in the previous case. Given Figure 3, we add e_a^3 and e_m^3 denoted by R_3R_3 and MHH_3 respectively. Note that the certainty equivalent of the less risk averse individual is greater than that of the more risk averse individual. The position of n_i is determined where R_iR_i intersects MHH_i, equation (11).

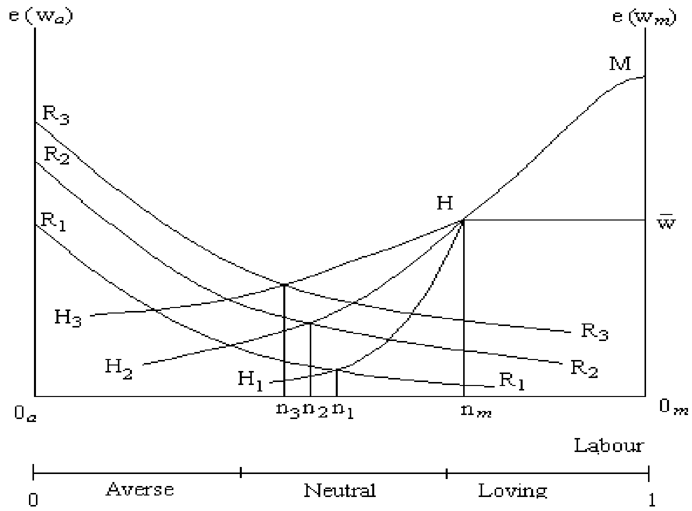

Figure 4: Geometric exposition of migration equilibrium in the three-type individual model, expressed in terms of certainty equivalent.

Figure 4 represents a migration equilibrium in the three-type individual model where $n_3 < n_2 < n_1$.

Following the same logic as in the simple two-type case, since $n_3 < n_2 < n_1$, we can predict that type 3 will be the first group of people who are willing to migrate to the urban areas followed by type 2. Therefore type 3 are more likely to stay in the urban areas while type 1 are likely to remain in the rural areas. As for type 2, their locations will depend on the position of n^* which depends on $\lambda_i, i = 1, 2, 3$.

We start with the case where $\lambda_1 + \lambda_2 \leq n_3$. Both types 1 and 2 will strictly prefer the rural areas and n^* will be equal to n_3. Thus type 3 will be indifferent between staying in either regions. If the strict inequality holds, a portion of type 3 equal to $(n_3 - (\lambda_1 + \lambda_2))$ will also be in the rural areas.

If $n_3 < \lambda_1 + \lambda_2 \leq n_2$, n^* will be exactly equal to $\lambda_1 + \lambda_2$. Both types 1 and 2 strictly prefer the rural areas while type 3 strictly prefer the urban areas. Note that if $n_2 = \lambda_1 + \lambda_2$, n^* is equal to n_2 and the type 2 are indifferent between staying in either regions.

If $n_2 < \lambda_1 + \lambda_2 \leq n_1$ and $\lambda_1 \leq n_2$, n^* will be equal to n_2. Some of type 2 $(= (\lambda_1 + \lambda_2) - n_2)$ and all of type 3 will be in the urban areas while all type 1 and the rest of type 2 will be in the rural areas.

If $n_2 < \lambda_1 \leq n_1$, n^* will be equal to λ_1. Only type 1 will be in the rural areas.

Finally if $n_1 < \lambda_1$, n^* will be equal to n_1. Only type 1 will be in the rural areas. There will also be $(\lambda_1 - n_1)$ of type 1 in the urban areas.

Suppose that $\lambda_i = \frac{1}{3}, i = 1, 2, 3$, as indicated in Figure 4, n^* is equal to n_2. In equilibrium, all type 1 and a portion of type 2 equal to $(n_2 - \lambda_1)$ will be in the rural areas, the rest of type 2 and all type 3 will be in the urban areas.

From the geometric exposition, n_1, n_2 and n_3 can be of any order depending on the values of variables and parameters such as those determining the production functions in both sectors, $\bar{w}, \pi$ and $\gamma_i, i = 1, 2, 3$. The migration equilibrium always exists. Without considering the proportions of individuals' types (λ_i), the positions of n_i tell us type i's preference towards different locations. If $n_h \leq n_j, h \neq j$, type h will be more willing to migrate to the urban areas than type j. However the migration may take place only when the potential migrant expects that his certainty equivalent in the urban areas is at least as high as his certainty equivalent in the rural areas.

Nonetheless the positions of n_i might not be monotonic leading to an equilibrium where both the risk averse and the risk loving are in one sector while the risk neutral are in another sector.

4 Some Remarks on Migration Decision

In this section, we focus on two possibilities that seem to be unusual for the migration model like the H-T model. First is the possibility of both the risk averse and risk loving individuals migrating to the urban areas. It is unusual that both types whose preferences towards risk are very different will choose the same choice. This may be because migration decision made by one type of individuals affect the other types via changes in the level of rural wage and urban employment probability (both depend on the level of total rural labour force). In other words, there are two forces driving our model: the average income and the regional risk. Any change in these two forces affect the values of certainty equivalent across sectors. Note also that the curvature and slope of certainty equivalent depend on the variables and parameters of the model. Since each individual makes migration decision by comparing his own certainty equivalent across sectors, it is possible that, in equilibrium, the order of n_i implies that we may find both the risk loving and risk averse in the urban areas.

Second, it is possible that a policy, aimed to use as a migration control, may lead to flows of migrants in both directions (out of and into the rural areas). This may create significant social and economic costs although the level of urban labour force may be lower.

4.1 A possibility that the risk averse and the risk loving migrate to the urban areas

Consider the following case where $n_1 < n_3 < n_2$, drawn in Figure 5: type 1 will be the first who migrate to the urban areas followed by type 3. Given λ_i, it is possible that, in equilibrium, there is a mixture of types 1 and 3 in the urban areas. Suppose that $\lambda_i = \frac{1}{3}, i = 1, 2, 3 : n^*$ will be equal to n_3. All type 2 and

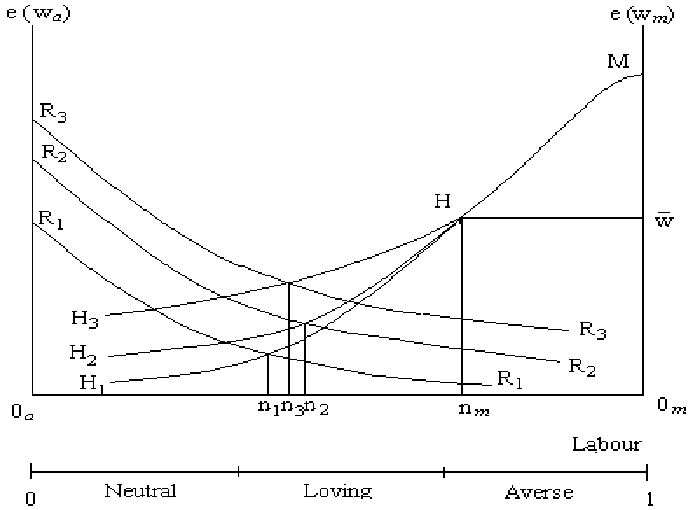

Figure 5: Geometric exposition of migration equilibrium in the three-type individual model, expressed in terms of certainty equivalent, where $n_1 < n_3 < n_2$.

a portion of type 3 equal to $(n_3 - \lambda_2)$ will be in the rural areas and the rest of type 3 and all type 1 will be in the urban areas.

Can a situation like the one described by Figure 5 occur in the LDCs? Theoretically, it is possible. We offer two explanations for this. First, a cause of migration is a lack of opportunity to earn income in the rural areas, Todaro and Smith (2003). Suppose that such lack of opportunity is a result of a poor infrastructure which cannot mitigate problems caused by having a bad condition for agricultural production. Therefore there exists a high risk in living in the rural areas in terms of earned income. On the other hand, as the manufacturing sector grows, there are a lot of job opportunities in the urban areas. If the unemployment rate is not too high, a risk averse individual might consider the manufacture as a relatively low risk sector and migrate to the urban areas.

However the unemployment rate in the urban areas in many major LDCs' cities is quite high, Todaro and Smith (2003). In addition, LDCs governments' policies have helped towards agricultural production, see Coxhead and Plangpraphan (1998) for evidence in Thailand. Thus we offer another explanation which involves the existence of the informal sector. If we allow the unemployed to earn income by working in the informal sector, this will lower the risk in the urban areas. Thus a risk averse individual may migrate to the urban areas if he views that risk in terms of earned income is relatively higher in the rural areas. Moreover there is evidence that some potential migrants migrate to the urban areas for the purpose of working in the informal sector (see TDRI, 1992). We consider the effect of the existence of informal sector on migration equilibrium analytically and numerically. The latter is presented in Example 1.

Suppose that an unemployed earns income in the informal sector which is assumed to be equal to a fraction of the minimum wage: that is the informal sector income is $c\bar{w}$ where $0 \leq c < 1$. The expected utility of an individual i in the urban areas, equation (4), and his certainty equivalent in the urban areas become

$$V_m^i = \left(\frac{n_m}{1-n^*}\right)\frac{\bar{w}^{1-\gamma_i}}{1-\gamma_i} + \left(1-\frac{n_m}{1-n^*}\right)\frac{(c\bar{w})^{1-\gamma_i}}{1-\gamma_i}$$

and

$$e_m^i = \left[\left(\frac{n_m}{1-n^*}\right)\bar{w}^{1-\gamma_i} + \left(1-\frac{n_m}{1-n^*}\right)(c\bar{w})^{1-\gamma_i}\right]^{\frac{1}{1-\gamma_i}}. \quad (12)$$

If the economy consists of type i only, n_i is defined when $e_a^i = e_m^i$:

$$\begin{aligned} & \left[(1-\pi)\left(w_{al}^i\right)^{1-\gamma_i} + \pi\left(w_{ah}^i\right)^{1-\gamma_i}\right]^{\frac{1}{1-\gamma_i}} \\ = & \left[\left(\frac{n_m}{1-n_i}\right)\bar{w}^{1-\gamma_i} + \left(1-\frac{n_m}{1-n_i}\right)(c\bar{w})^{1-\gamma_i}\right]^{\frac{1}{1-\gamma_i}} \quad (13) \end{aligned}$$

where $w_{a\theta}^i = f_{a\theta}'(n_i), \theta = l, h$.

Compare expressions of e_m^i in (12) and (9), the possibility of earning income in the informal sector shifts e_m^i upwards. However magnitude of the shift in the HH_i curve will not be the same for all i, it will depend on the value of γ_i. Starting with $c = 0$, suppose that the migration model is the one drawn in Figure 4. Allowing $c > 0$, it is possible that, in the new equilibrium, the economy is the one represented by Figure 5: that is both type 1 and type 3 may be found in the urban areas. We give a numerical example of such possibility in Example 1. The values of the data used in our examples are taken from Krichel and Levine (1999) where they consider an extended H-T model with flexible urban wage.[1]

Assume that the production functions in both sectors take the form of Cobb-Douglas production function:

$$\begin{aligned} Y_m &= Bn_m^\beta, B > 0, \beta \leq 1 \\ Y_a &= An_a^\alpha, A > 0, \alpha \leq 1 \end{aligned}$$

where β and α are the shares of labour in manufacturing and agricultural productions respectively. Note also that the agriculture and manufacture terms of

[1]Krichel and Levine (1999) find that in the absence of other instruments, such as employment subsidies, laissez-faire migration is excessive unless the urban real wage is sufficiently responsive to unemployment. Moreover they show that the laissez-faire migration is too low compared with the first-best outcome supported by a pair of employment subsidies, if financing the subsidies involves no costs. Nevertheless, by simulations based on data of India, such programme requires too high level of income tax increase which generates economic costs including collection costs and global supply-side effects. Therefore the optimal subsidy and taxation policy falls far short of the first-best outcome.

trade is equal to 1. Assume further that

$$\begin{aligned} Y_{al} &= \phi Y_a \\ Y_{ah} &= Y_a, \phi < 1 \end{aligned}$$

where ϕ is the fraction of total agricultural products that can be produced under a bad condition. Therefore the urban and rural productivities are

$$\begin{aligned} w_m &= \beta B n_m^{\beta-1} \\ w_a &= \alpha A n_a^{\alpha-1} \end{aligned}$$

and

$$\begin{aligned} w_{al} &= \phi w_a \\ w_{ah} &= w_a. \end{aligned}$$

The data, taken from Krichel and Levine (1999), that are relevant to our numerical examples are given in Table 1, where u is the unemployment rate, $u = \frac{1-n_a-n_m}{1-n_a}$,

Table 1: Data on Variables and Parameters of the Model

Variable	Value	Source
w_m/w_a	1.3-1.4	Government of India (1991)
u	0.11-0.12	Government of India (1991)
β	0.6-0.8	Ahluwalia (1985)
α	0.5	Shukla and Stark (1990)
n_m/n_a	0.33	Government of India (1991)

note also that these values are derived in equilibrium. The assumption of flexible urban wage in Krichel and Levine (1999) does not affect the labour market mechanism of the H-T model. The only important assumption is that the urban wage is higher than the market clearing wage which may be because of efficiency wage (Stiglitz, 1976) or the imposition of the minimum wage which is the most convenient tool. Therefore for convenience and consistency to our model, we set the urban wage in Table 1 to be equal to the minimum wage.

Example 1 *Suppose that there are three types of individuals: the risk loving (type 3), the risk neutral (type 2), and the risk averse (type 1). Since the values of data in Table 1 are derived in equilibrium, we assume that the ratios hold if the economy consists of only type i individuals. Assuming that* $\bar{w} = w_m$*, we set*

$$\bar{w} = 1.4 w_a \text{ and } n_m = 0.33 n_i$$

where n_i *is the equilibrium level of rural labour force if the economy consists of type i only. Consider also a possibility that the urban unemployed can earn income from the informal sector. From* (13) *,substituting in the values of* $\bar{w}, w_{al}, w_{ah}$ *and* n_m*:*

$$(1-\pi)\,\phi^{1-\gamma_i} + \pi = \frac{0.33 n_i}{1-n_i}\,(1.4)^{1-\gamma_i} + \left(1 - \frac{0.33 n_i}{1-n_i}\right)(1.4c)^{1-\gamma_i}. \tag{14}$$

Setting $\pi = 0.75, \phi = 0.4, \gamma_3 = -0.5, \gamma_2 = 0, \gamma_1 = 0.5,$ *the values of* n_i *defined in* $(14), i = 1, 2, 3,$ *are given below*

	$c = 0$	$c = 0.5$
n_1	0.699	0.38
n_2	0.648	0.394
n_3	0.598	0.39

When $c = 0, n_3 < n_2 < n_1,$ *as drawn in Figure 4. The agricultural sector is considered to have a relatively lower risk in earned income compared to the manufacturing sector. The type 3 are the first group of people who are willing to leave the rural areas followed by the type 2. Nevertheless when* $c = 0.5, n_1 < n_3 < n_2,$ *as drawn in Figure 5. The risk in the urban areas is reduced by the possibility of earning income from the informal sector. The order of* n_i *has changed. The type 1 will be the first who are willing to leave the rural areas followed by the type 3. Suppose that* $\lambda_i = \frac{1}{3}, i = 1, 2, 3.$ *In the initial equilibrium,* $n^* = 0.648:$ *all type 1 are in the rural areas, all type 3 are in the urban areas, and type 2 are divided between the two regions where most of type 2 are in the rural areas. In the new equilibrium,* $n^* = 0.39:$ *all type 2 are in the rural areas while all type 1 are in the urban areas. The type 3 are divided between the two regions. Note that the presence of the informal sector leads to more rural-urban migration. The level of rural labour force in the new equilibrium is lower. This is what we expect to happen when the expected urban wage (utility) increases, and the urban risk reduces.*

4.2 A possibility of migration in both directions

Given a migration model drawn in Figure 5. Suppose that the government aims to control the level of migration by imposing a policy that reduces the rural uncertainty in terms of earned income. That is the policy increases the level of agricultural production under a bad condition. Thus the rural income increases. As predicted in the H-T model, there will be a reverse flow of migration back to the rural areas. In our model, this may lead to a change in the order of n_i and hence migration in both directions. We can also use a numerical example to show such possibility.

Example 2 *Let the economy be the one described in Example 1 where there exists the informal sector,* $c > 0.$ *Given the same values of* $\pi, \phi, \gamma_i, i = 1, 2, 3,$ *the values of* n_i *shown in Example 1 are*

$$n_1 = 0.38, n_2 = 0.394, n_3 = 0.39$$

that is the order of n_i *is*

$$n_1 < n_3 < n_2.$$

Suppose that the government imposes a policy which increases the agricultural production under the bad condition such that $\phi = 0.5.$ *Setting the values of* π

and $\gamma_i, i = 1, 2, 3,$ *as in Example 1, the values of* n_i *become*

$$n_1 = 0.44, n_2 = 0.43, n_3 = 0.42$$

that is the order of n_i *is*

$$n_3 < n_2 < n_1.$$

Suppose also that $\lambda_i = \frac{1}{3}, i = 1, 2, 3$. *In the initial equilibrium, Figure 5,* $n^* = 0.39$, *all type 2 are in the rural areas, all type 1 are in the urban areas, and type 3 are divided between the two areas. In the new equilibrium, Figure 4,* $n^* = 0.43$, *all type 1 are in the rural areas, all type 3 are in the urban areas, and type 2 are divided between the two areas. Although the new equilibrium level of urban labour force is lower than the initial equilibrium level, the process involves massive flows of migration in both directions. This may generate social and economic costs which cannot be explicitly modelled in our analysis. The costs may outweigh the benefit of having low level of urban unemployment or lower level of informal sector workers.*

4.3 Policy implications

Some policy implications can be drawn from the above examples. First if the migration model is represented by Figure 4, a policy that reduces the risk in the rural areas will not affect the order of n_i but will increase the level of total rural labour force. Thus it may be used as a policy for migration control. Examples of such policy are rural employment subsidy, rural investment that increases agricultural production, and the creation of rural insurance markets against the bad condition. The first two policies have been suggested in the H-T literature as policies for migration control, see Harris and Todaro (1970), Bhagwati and Srinivasan (1974), and Basu (1980). The creation of rural insurance markets has been mentioned in Katz and Stark (1986) where the cause of rural-urban migration of a risk averse individual is the imperfection of rural money (capital) markets.

Second, a policy for migration control may not be desirable as it seems at first sight if it leads to a change in the order of n_i which leads to migration in both directions. Nevertheless, Example 2 also implies that it may be possible that there exists boundary of the values of variables and parameters such that the policy may not change the order of n_i. Therefore the policy for migration control may still be desirable given that it is carefully implemented within an appropriate boundary.

5 Conclusion

We have considered a migration model in which there exist regional differences and uncertainties. In the urban areas, the minimum wage has been imposed at the level higher than the market clearing wage leading to an excess supply of labour. The urban uncertainty is represented by the chance of becoming unemployed if an individual decides to migrate to the urban areas. On the other hand, the rural uncertainty is caused by random states of nature. Thus if an individual decides to stay in the rural areas, he faces the uncertain level of agricultural outputs. The assumption of rural uncertainty is reasonably realistic in a developing country with primary agricultural technology. The cause of rural uncertainty may be different from our assumption. Katz and Stark (1986) assume that such uncertainty is caused by imperfection of money (capital) markets in the rural areas. However it also leads to uncertain level of agricultural output,[2] therefore the labour market mechanism will be the same. That is a potential migrant compares his certainty equivalent across sectors and migrates to the sector that gives him the highest certainty equivalent. The advantage of using the certainty equivalent, which is derived from the expected utility theory, is that it has monetary unit and it allows us to compare our migration model directly to the H-T model.

To derive the equilibrium labour allocation, we need to assume that individuals can be ranked in terms of certainty equivalent. Moreover we need an extra information on the proportions of each type of individuals. The equilibrium always exists and the individual at the margin is indifferent between staying in either regions. The level of rural labour force and the distribution of individuals' types across sectors in equilibrium depend on the variables and parameters of the model such as those determining the production functions in both sectors, the level of the minimum wage, the probability of having a good condition for agricultural production, and the individuals' degrees of risk aversion. Thus the equilibrium labour allocations may not be monotonic with respect to degrees of risk aversion. We may have an equilibrium where both the risk loving and risk averse individuals migrate to the same sector. We use a numerical example, Example 1, to show such possibility. Therefore a risk averse individual migrates to the urban areas if he views that the risk associated with the urban areas is relatively lower than the risk associated with the rural areas.

The possibility that both the risk loving and risk averse are in the urban areas affects authorities' decisions to implement different policies to mitigate problems of urban unemployment. For example, a policy that reduces the rural uncertainty will increase the rural income and will attract the risk averse back to the rural areas whereas it will induce other types that initially remain in the rural areas to migrate to the urban areas. This leads to migration in both directions, see Example 2. Thus the policy that seems, at first sight, to be a desirable tool for migration control may involve too high social and economic costs. However to prevent migration in both directions, the policy should be implemented within the appropriate boundary which depends on the model's variables and parameters. Although the examples are given by assuming the values of the parameters of the model, the analysis draws attention to both the

[2] For example, those who have ability to borrow from the formal money market are able to invest on land or other inputs that would lead to higher output thus high income. However those who cannot borrow from the formal money market may turn to the informal money market. It is likely that a significant portion of their income will have to be used to pay back the unreasonable high interest rate. Thus they are left with a low level of income.

type of appropriate policy and its level.

Thus our model generalises the H-T model in such a way that it offers an explanation of how heterogeneous individuals allocate themselves across sectors where both regional differences and uncertainties exist. The model highlights the role of risk in migration decision, and capability of policy application in terms of policies' types and their boundaries.

References

[1] Ahluwalia, I. (1985) *Indian Industrial Performance*. Delhi: Oxford University Press.

[2] Basu, K. (1980) "Optimal Policies in Dual Economies" *The Quarterly Journal of Economics*, vol. 95, issue 1, p187-196.

[3] Becker, R. A. (2001) "Comparing risk aversion in a probability triangle" *Economic Theory*, vol. 17, p739-752.

[4] Bernoulli, D. (1954) "Exposition of a New Theory on the Measurement of Risk" (original 1738) *Econometrica*, vol. 22, p23-36.

[5] Bhagwati, J.N., and Srinivasan, T.N. (1974) "On reanalyzing the Harris-Todaro model: Policy rankings in the case of sector-specific sticky wages" *American Economic Review*, vol. 64, p502-508.

[6] Corden, W.M., and Findlay, R. (1975) "Urban Unemployment, Intersectoral Capital Mobility and Development Policy" *Economica*, Feb. 1975, p59-78.

[7] Coxhead, I., and Plangpraphan, J. (1998) "Thailand's economic boom and agricultural bust: Some economic questions and policy puzzles" University of Wisconsin-Madison, Department of Agricultural and Applied Economics, Staff Paper Series No. 419 (September 1998).

[8] Ghatak, S., Levine, P., and Wheatley Price, S. (1996) "Migration Theories and Evidence: An Assessment" *Journal of Economic Surveys*, vol. 10, no. 2, p159-198.

[9] Government of India (1991) *Statistical Abstract*. Delhi: C.S.O.

[10] Harris, J. and Todaro, M. (1970) "Migration, Unemployment and Development: A Two-Sector Analysis" *American Economic Review*, vol. 60, p126-142.

[11] Hirshleifer, J., and Riley, J. G. (1997) The analytics of uncertainty and information. Cambridge University Press.

[12] Katz, E., and Stark, O. (1986) "Labour Migration and Risk Aversion in Less Developed Countries", *Journal of Labor Economics*, vol. 4, no. 1, p134-149.

[13] Katz, E., and Stark, O. (1987) "International Migration under Asymmetric Information", *The Economic Journal*, vol. 97, no. 1, p718-726.

[14] Krichel, T., and Levine, P. (1999) "The Welfare Economics of Rural-to-Urban Migration: The Harris-Todaro Model Revisited", *Journal of Regional Science*, vol. 39, no. 3, p429-447.

[15] Lewis, W. A. (1954) "Economic Development with Unlimited Supplies of Labour", *The Manchester School*, vol. 22, p139-191.

[16] Machina, M. J. (1987) "Choice under Uncertainty: Problems Solved and Unsolved", *Journal of Economic Perspectives*, vol. 1, p121-154.

[17] Mas-Colell, A., Whinston, M. D., and Green, J. R. (1995) Microeconomic Theory. Oxford: Oxford University Press.

[18] Von Neumann, J., and Morgenstern, O. (1947) The Theory of Games and Economic Behaviour, Second Edition. Princeton: Princeton University Press.

[19] Pratt, J. W. (1964) "Risk aversion in the small and large" *Econometrica*, vol. 32.

[20] Shukla, V., and Stark, O. (1990) "Policy Comparisons with an Agglomeration Effects Augmented Dual Economy Model" *Journal of Urban Economics*, vol. 27, p1-27.

[21] Stark, O. (1980) "On the Role of Urban-to-Rural Remittances in Rural Development" *Journal of Development Studies*, p191-196.

[22] Stark, O. (1984) "Rural-to-Urban Migration in LDCs: A Relative Deprivation Approach" *Economic Development and Cultural Change*, vol. 32, April (1984), p475-486.

[23] Stark, O. (1991) The Migration of Labor. Basil Blackwell.

[24] Stark, O. and Bloom, D. E. (1985) "The New Economics of Labor Migration" *The American Economic Review*, vol. 75, issue 2, p173-178.

[25] Stark, O. and Levhari, D. (1982) "On Migration and Risk in LDCs" *Economic Development and Cultural Change*, vol. 32, April (1984), p475-486.

[26] Starmer, C. (2000) "Developments in Non-Expected Utility Theory: The Hunt for a Descriptive Theory of Choice under Risk" *Journal of Economic Literature*, vol. XXXVIII, p332-382.

[27] Stiglitz, J.E. (1976) "The Efficiency Wage Hypothesis, Surplus Labour, and the Distribution of Income in L.D.C.s" *Oxford Economic Papers*, p185-207..

[28] Thailand Development Research Institute Foundation (TDRI) (1992) "Managing the Urban Informal Sector in Thailand: A Search for Practical Policies Based on the Basic Minimum Needs Approach", March 1992.

[29] Todaro, M. P. (1969) “A Model of Labor Migration and Urban Unemployment in Less Developed Countries” *American Economic Review*, vol. 59, p.138-148.

[30] Todaro, M. P., and Smith, S. C. (2003) Economic Development, Eight Edition. Pearson Education Limited.

[31] Varian, H. R. (1992) Microeconomic Analysis, Third Edition. W. W. Norton & Company, Ltd.

[32] Xu, C. (1992) Risk Aversion, Rural-Urban Wage Differentiation And Migration, Discussion Paper No. 108: *CEPR*; London.

6 Appendix A: The Harris-Todaro Model

Define n_{urb}, n_m, and n_a as proportions of total urban labour force, urban employment and rural employment, respectively. Since there is no rural unemployment, n_a is also proportion of total rural labour force. Labour markets in both sectors are perfectly competitive. Assume also that the economy is a small-open economy. Thus relative prices of the manufacturing and agricultural outputs are fixed and given by the world market. The price ratio between the respective outputs is normalised to 1.

The total labour force is also normalised to 1. The labour endowment constraint is

$$n_{urb} + n_a = 1. \tag{15}$$

The agricultural output (Y_a) is determined by

$$Y_a = f_a(n_a) := F_a\left(n_a, \bar{L}, \bar{K}_a\right) \tag{16}$$

where $\bar{L}$ and $\bar{K}_a$ are fixed amounts of land and capital respectively. Hence n_a is the only variable input. $f_a' > 0$ and $f_a'' < 0$ $\left(f_a' = \frac{df_a}{dn_a} \text{ and } f_a'' = \frac{df_a'}{dn_a}\right)$. The rural wage is the marginal product of labour which is

$$w_a = f_a'(n_a). \tag{17}$$

Similarly, the manufacturing output (Y_m) is determined by

$$Y_m = f_m(n_m) := F_m\left(n_m, \bar{K}_m\right) \tag{18}$$

where $\bar{K}_m$ is fixed capital. Hence n_m is the only variable input and $f_m' > 0, f_m'' < 0$. Since the labour market is perfectly competitive, each worker is employed at his marginal productivity:

$$\bar{w} = f_m'(n_m). \tag{19}$$

Thus n_m can be written as a function of the minimum wage $(\bar{w})$:

$$n_m = f_m'^{-1}(\bar{w}). \tag{20}$$

A potential migrant moves to the urban areas as long as the expected urban wage is higher than the rural wage. The equilibrium is reached when

$$w_a = \frac{n_m}{(1-n_a)}\bar{w}. \tag{21}$$

The right hand side of (21) is the expected urban wage which is equal to the product of the probability of obtaining an urban job (the ratio of the number of urban jobs and the total urban labour force) and the minimum wage. The left hand side of (21) is the expected rural wage which is the real rural wage.

In equilibrium, the level of urban unemployment (U) is

$$U = 1 - n_a - n_m \tag{22}$$

which is greater than zero as long as there exist sectoral wage differentials. Given (21),

$$U = \left(\frac{\bar{w}}{w_a} - 1\right) n_m. \tag{23}$$

And the unemployment rate is defined as

$$u = \frac{U}{(1-n_a)}. \tag{24}$$

Given (22) and (24), the following expression is also useful:

$$n_m = (1-u)(1-n_a). \tag{25}$$

The existence of urban unemployment can be seen clearly by using the geometric exposition of the H-T model introduced by Corden and Findlay (1975).

6.1 Geometric Exposition of the Harris-Todaro Model

The H-T model assumes a fixed number of total labour force which we normalise to 1 and is represented by the length 0_a0_m in Figure 6. The MM curve represents the marginal value product of labour in the manufacturing sector (MPL_m) measured at the externally given world prices and is drawn with origin 0_m. The AA curve represents the marginal value product of labour in the agricultural sector (MPL_a) measured also at the given world prices and is drawn with origin 0_a. In the standard competitive model, the intersection of the two curves at E determines the equilibrium employment levels. The equilibrium wage is uniform and is equal to EL_0. There are 0_aL_0 and 0_mL_0 employees in the agricultural and manufacturing sectors respectively.

As a result of imposing the minimum wage above the market-clearing wage, E cannot be attained. The urban employment is 0_mn_m. The part of the MM curve beyond the point $(n_m, \bar{w})$, starting from the origin 0_m, is replaced by the rectangular hyperbola, the HH curve. This curve represents the expected marginal value product of labour in the manufacturing sector. It could lie above or below the MM curve depending on urban wage elasticity of labour.

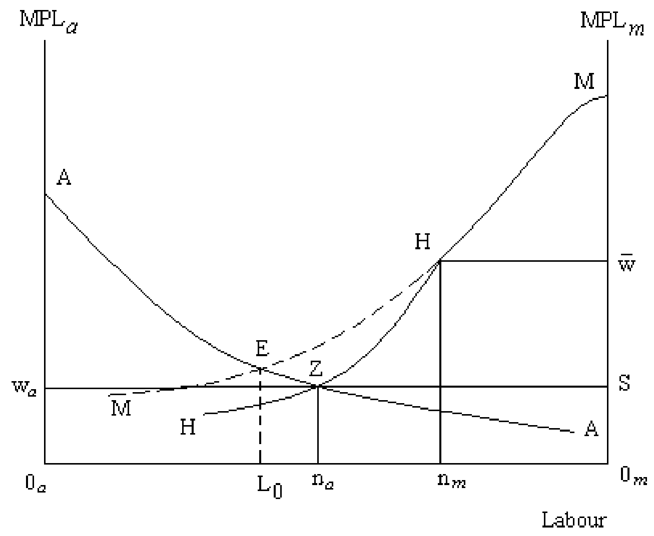

Figure 6: Geometric Exposition of the Harris-Todaro Equilibrium

7 Appendix B: Application of the Expected Utility Theory

We present here a brief discussion of the expected utility theory, for a more detailed discussion see Mas-Colell, Whinston, and Green (1995); Hirshleifer and Riley (1997); and Starmer (2000).

Define a lottery $\mathbf{p}$ as a probability distribution $\mathbf{p} = (p_1, ..., p_S)$ over a fixed set of outcomes $\mathbf{X} = (x_1, ..., x_S)$ where p_s is the probability of x_s, $p_s \geq 0$ for all $s = 1, ..., S$ and $\sum_s p_s = 1$.[3] Note that in our model, the outcomes take the form of monetary payoffs. Denote $\mathcal{L}$ to be the set of all lotteries over the set of outcomes, $\mathbf{X}$. We state definitions without proofs of the three axioms of the expected utility theory which are ordering; continuity; and independence.

Definition 1 *The preference relation* $\succsim$ *on the space of lotteries* $\mathcal{L}$ *is* ordering *if it satisfies the following properties, for any* $\mathbf{p}, \mathbf{q}, \mathbf{r} \in \mathcal{L}$,

(i) completeness: *either* $\mathbf{p} \succsim \mathbf{q}$ *or* $\mathbf{q} \succsim \mathbf{p}$ *or both*; *and*

(ii) transitivity: *if* $\mathbf{p} \succsim \mathbf{q}$ *and* $\mathbf{q} \succsim \mathbf{r}$, *then* $\mathbf{p} \succsim \mathbf{r}$.

Definition 2 [Mas-Colell, Whinston, and Green (1975)] *The preference relation* $\succsim$ *on the space of lotteries* $\mathcal{L}$ *is* continuous *if for any* $\mathbf{p}, \mathbf{q}, \mathbf{r} \in \mathcal{L}$, *the*

[3] In the expected utility theory literature, N is often used as a notation representing the number of possible outcomes. However we already use N to represent total number of population. Thus, to avoid any confusion, S will be used to represent the total number of possible outcomes.

set

$$\{\delta \in [0,1] : \delta \mathbf{p} + (1-\delta)\,\mathbf{q} \succsim \mathbf{r}\} \subset [0,1]$$

and

$$\{\delta \in [0,1] : \mathbf{r} \succsim \delta \mathbf{p} + (1-\delta)\,\mathbf{q}\} \subset [0,1]$$

are closed.

Definition 3 [Mas-Colell, Whinston, and Green (1975)] *The preference relation* $\succsim$ *on the space of lotteries* $\mathcal{L}$ *satisfies* the independence axiom *if for any* $\mathbf{p}, \mathbf{q}, \mathbf{r} \in \mathcal{L}$ *and* $\delta \in [0,1]$, *we have*

$$\mathbf{p} \succsim \mathbf{q} \textit{if and only if } \delta \mathbf{p} + (1-\delta)\,\mathbf{r} \succsim \delta \mathbf{q} + (1-\delta)\,\mathbf{r}.$$

Assume that the decision maker has a rational preference relation $\succsim$ on the set of all lotteries $\mathcal{L}$ which satisfies all three axioms. Then his preferences can be represented by a utility function with the expected utility form:

$$V(\mathbf{p}) = \sum_s p_s v(x_s)\,, i = 1, ..., N \tag{26}$$

where $v(\cdot)$ is a utility function defined on the set of consequences. We call $V(\cdot)$ the *von Neumann-Morgenstern* expected utility function, Neumann and Morgenstern (1947).

Assume that the outcomes are amounts of money which, for convenience purpose, can be treated as continuous variable. Let $G : \mathcal{R} \to [0,1]$ be a cumulative distribution function of the lottery. Thus for any x, $G(x)$ is the probability that the realised payoff is less than or equal to x. Then the equation (26) can be written as

$$V(G) = \int v(x)\, dG(x)\,. \tag{27}$$

As in Mas-Colell, Whinston, and Green (1975), for notation simplicity, we do not explicitly write the limits of integration when the integral is over the full range of possible realisations of x. Assume further that $v(\cdot)$ is monotonically increasing, continuous, and twice differentiable.

In our model, the lottery in the urban areas is a probability distribution of employment $\mathbf{p_m} = \left(1 - \frac{n_m}{1-n_a}, \frac{n_m}{1-n_a}\right)$ over monetary outcomes $\mathbf{X_m} = (0, \bar{w})$. On the other hand, the lottery in the rural areas is $\mathbf{p_a} = (1-\pi, \pi)$ over monetary outcomes $\mathbf{X_a} = (w_{al}, w_{ah})$ where π is the probability of having a good condition for agricultural production.

Assume that the three axioms of the expected utility theory hold, the expected utilities in the urban and rural areas are respectively defined as

$$V(\mathbf{p_m}) = \left(1 - \frac{n_m}{1-n_a}\right) v(0) + \frac{n_m}{1-n_a} v(\bar{w})\,, \tag{28}$$

$$V(\mathbf{p_a}) = (1-\pi)\, v(w_{al}) + \pi v(w_{ah})\,. \tag{29}$$

Individuals migrate to maximise their expected utilities. Therefore the migration equilibrium is defined where $V(\mathbf{p_a}) = V(\mathbf{p_m})$:

$$(1-\pi)\, v(w_{al}) + \pi v(w_{ah}) = \left(1 - \frac{n_m}{1-n_a}\right) v(0) + \frac{n_m}{1-n_a} v(\bar{w}) . \tag{30}$$

To establish migration pattern, in other words who undertakes the migration decision, the crucial assumption is that we can rank individuals with respect to their preferences towards risk. Moreover we need to assume a functional form of $v^i(\cdot)$ but, first, we consider the concept of risk aversion.

7.1 Risk Aversion, Neutrality and Loving

Recall that $v(\cdot)$ is monotonically increasing and twice differentiable, the first definition of risk aversion in terms of the utility function is given as follows:

Definition 4 [Mas-Colell, Whinston, and Green (1975)] *A decision maker is* risk averse *if for any lottery $G(\cdot)$, the degenerate lottery that yields the amount $\int x dG(x)$ with certainty is at least as good as the lottery $G(\cdot)$ itself. If the decision maker is always* [*i.e., for any $G(\cdot)$*] *indifferent between these two lotteries, we say that he is* risk neutral. *Finally, we say that he is* strictly risk averse *if indifference holds only when the two lotteries are the same* [*i.e., when $G(\cdot)$ is degenerate*].

From Definition 4, the decision marker is risk averse if and only if

$$\int v(x)\, dG(x) \leq v\left(\int x dG(x)\right) \tag{31}$$

for all $G(\cdot)$. In the context of expected utility theory, this is equivalent to the concavity of $v(\cdot) : v'(\cdot) > 0$ and $v''(\cdot) < 0$, which can be shown using a graphical illustration drawn in Figure 7.

From Figure 7, a lottery $G(\cdot)$ offers either x_1 or x_2 with some probabilities p_1 and $p_2 = 1 - p_1$ respectively. The expected value of these outcomes is $\int x dG(x)$ at which the lottery yields the expected utility of $\int v(x)\, dG(x)$, the point C on the chord AB. By Definition 4, a risk averse decision maker prefers the lottery that yields the amount $\int x dG(x)$ with certainty over the lottery $G(\cdot)$ itself. That is the utility function of this individual must be concave. The utility of the sure bet offering $\int x dG(x)$ is represented by the point D which is clearly higher than the point C.

Analogously, the decision maker is risk neutral if and only if $\int v(x)\, dG(x) = v\left(\int x dG(x)\right)$ for all $G(\cdot)$. Moreover he is risk loving if and only if $\int v(x)\, dG(x) \geq v\left(\int x dG(x)\right)$ for all $G(\cdot)$, which is equivalent to the convexity of $v(\cdot) : v'(\cdot) > 0$ and $v''(\cdot) > 0$.

The following concepts are useful when we compare risk aversion across individuals.

Definition 5 [Mas-Colell, Whinston, and Green (1975)] *Given a utility function $v(\cdot)$, the* certainty equivalent *of $G(\cdot)$ denoted by $e := e(G, v)$, is*

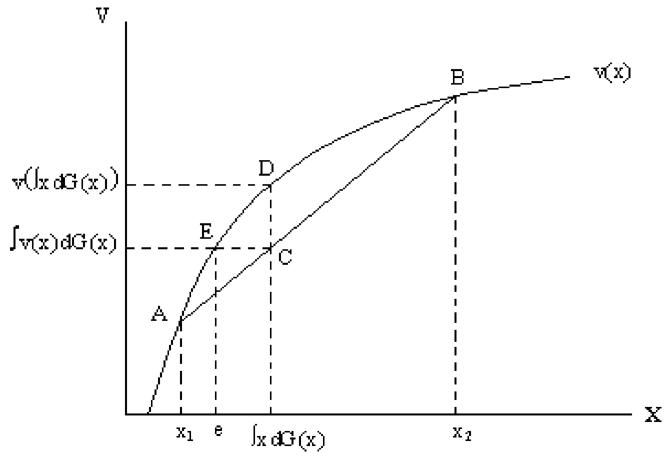

Figure 7: Utility function exhibiting risk aversion

the amount of money for which the individual is indifferent between the gamble $G(\cdot)$ *and the certain amount* e; *that is*

$$v(e) = \int v(x)\, dG(x). \tag{32}$$

From Figure 7, the certainty equivalent is the amount e which yields utility represented by the point E. The satisfaction of the inequality

$$e \leq \int xdG(x) \text{ for all } G(\cdot)$$

is equivalent to the decision maker being risk averse:

$$e \leq \int xdG(x)$$

$$\Leftrightarrow \quad v(e) \leq v\left(\int xdG(x)\right)$$

$$\Leftrightarrow \quad \int v(x)\, dG(x) \leq v\left(\int xdG(x)\right)$$

which is the same as condition (31). The following definition concerns the measurement of risk aversion.

Definition 6 *Given a twice-differentiable utility function $v(\cdot)$ for money, the Arrow-Pratt coefficients of absolute and relative risk aversion at x*, denoted by r_A and r_R respectively, *are defined as*

$$r_A(x) = -\frac{v''(x)}{v'(x)} \tag{33}$$

and

$$r_R(x) = -\frac{xv''(x)}{v'(x)}. \tag{34}$$

It follows from Definition 4 that for a risk averse decision maker, $r_A(x) > 0$ and $r_R(x) > 0$; for a risk loving decision maker, $r_A(x) < 0$ and $r_R(x) < 0$; and for a risk neutral decision maker, $r_A(x) = 0$ and $r_R(x) = 0$.