

**THE LEWIS MODEL**  
**Graphical Computer Applications**  
**for Kenya**

Michael Hodd<sup>1</sup>  
University of Westminster

Paper for Conference on  
***The Lewis Model after 50 Years:***  
***Assessing Sir Arthur Lewis's Contribution to Development Economics &***  
***Policy***  
6-7 July 2004.  
University of Manchester.

## 1 Introduction

This paper presents an 'Unlimited Labour' model (Lewis 1954) adapted to Kenya for illustrative purposes. The model runs in the Excel spreadsheet application, is readily adaptable to other developing countries, and allows the exploration of a variety of scenarios. The graphical displays show the main diagrams associated with the Lewis Model, as well as tracking wage rates, labour forces, saving rates and growth rates over time.

Nonetheless, the model presented here does not address numerous salient features of the development process covered in the original 'Unlimited labour' article. These include the identification of the traditional sector with agriculture; and the resulting terms of trade issues; the finance of investment by bank credit; introduction of a government sector; capital flows and migration; and the impact of international trade.

## 2 The Model

The model considers only a single-product economy. This means, in effect, that the modern sector produces agricultural output via a large-farm sector, as well as other outputs. This is quite appropriate for an application to Kenya.

The model is summarized in the Panel (1) of Figure 1, the printout of the Excel worksheet. The traditional sector produces output ( $Q_T$ ) from an input of labour ( $L_T$ ), and, in addition, benefits from a steady improvement in techniques, occurring at a rate  $q$ . The production function is of Cobb-Douglas form, but constrained to give unchanging levels of output for levels of labour input in excess of the level at which labour is regarded as being in surplus - see Panel (10) of Figure 1.

The modern sector produces output ( $Q_M$ ) from inputs of labour ( $L_M$ ) and capital ( $K_M$ ), and benefits from technical progress at rate  $r$ . Again the production function is Cobb-Douglas.

Investment ( $I$ ) is taken to be a constant fraction ( $s$ ) of profits ( $\Pi$ ) in the previous year. It is assumed that the labour force is growing at rate  $g$ , and that the labour force is also growing at this rate.

All these features are necessary to cover the main features of the Kenyan economy, although in the illustration of the model presented in this paper - to start with the simplest possible case - technical progress in both sectors is assumed to be zero, and there is no population growth.

### 3 Operation of the Model

Using the initial conditions specified in Panel (4), together with the parameters specified in Panel (3), the model calibrates the modern sector production function by calculating the constant term ( $\mathbf{A}_M$ ) to give the initial output. Similarly for the traditional sector and  $\mathbf{A}_T$ , except the initial output is related to the labour input less surplus labour. For labour inputs above this level, output is held constant - again see Panel (10).

The model calculates the average product in the traditional sector at the initial date, and this taken to be the wage (see Lewis 1954, p148). The marginal product of labour in the modern sector ( $\mathbf{MPL}_M$ ) is then calculated for the initial year, from the production function, and the difference between this and the traditional sector wage is taken to be the premium required for a traditional sector worker to move to the modern sector. The  $\mathbf{MPL}_M$  is taken as the supply price for labour to the modern sector until the 'turning point' is reached, when, as a result of labour being withdrawn from the traditional sector, the  $\mathbf{MPL}_T$  rises to the level of  $\mathbf{MPL}_M$ , less the premium - see Panel (11).

The expression for the  $\mathbf{MPL}_M$ , derived from the production function is inverted to give the quantity of labour demanded by the modern sector when the  $\mathbf{MPL}_M$  is equal to the constant supply price of labour (see above). This is shown in Panel (11), with the demand schedule, at successive time periods, shown shifting to the right as more capital is accumulated.

Assuming that factors are paid the value of their marginal products in the modern sector, the share of profits in output is given by the exponent of the capital term ( $\mathbf{c}_2$ ) in the modern sector production function. Given the level of output in the modern sector (given in the initial year, calculated from the production function thereafter), the level of profits ( $\mathbf{\pi}$ ) can be calculated. The given ratio of savings to profits ( $\mathbf{s}$ ) is used to calculate investment ( $\mathbf{I}$ ), which is added to the capital stock for the next year.

The constant supply price for labour ends when the withdrawal of labour from the traditional sector raises the  $\mathbf{MPL}_T$ , to the level of the  $\mathbf{MPL}_M$ , less the premium. Thereafter the model calculates the demand for labour in the modern sector such that the  $\mathbf{MPL}_M$  equals the  $\mathbf{MPL}_T$ . It is not possible to solve for the level of modern sector labour directly, and it has to be determined in the model by an iterative process. In this stage the supply schedule for labour in the modern sector ceases to be horizontal, and it has a positive slope, see Panel (11).

#### 4 Adaption to Kenya

As observed earlier, Kenya is a suitable case for presenting as a single-product application of the Lewis model, as it has a traditional sector and a modern large farm sector.

The parameters for the production functions come from Hodd 1974b. The estimated exponents for the production functions for large farm agriculture, industry and services were weighted by outputs to give the exponents for the modern sector production function, and are shown in Panel (3). The production function estimates all have significant rates of technical progress ( $q$  and  $r$ ), but for the current display these rates have been assumed to be zero. Similarly for the rate of population growth. The savings rate is taken as the ratio of investment to modern sector profits, the former derived from the capital stock figures in Hodd 1974b, and the latter calculated from the profit share given by the capital exponent in the modern sector production function.

The earliest set of consistent national accounts data for Kenya is for 1954 (such serendipity) and the exercise takes this as the start year. The data for initial outputs, labour forces and capital stocks come from Hodd 1974b.

It is assumed that 50% of the traditional labour force in 1954 was surplus - there is no direct evidence for this, but it probably doesn't do much violence to the facts - see Panel (10). The estimated traditional sector production function (Hodd 1974b) gives the  $MPL_T$  as £ K 2 per year (in constant 1964 prices - as are all the estimates in the model). The average product in the traditional sector, taken to be the wage, is £ K 22. If the estimated effects of technical progress and the weather are taken out of the figures for traditional sector output there is no positive trend to output over the period 1954 to 1972, when traditional sector labour was clearly rising due to quite rapid population growth, providing some further evidence of a zero  $MPL_T$ .

It is assumed that the traditional work force is 40% of the population in the traditional sector.

#### 5 Comments on the Kenyan Results.

The first feature that deserves comment is the difference between the wage in the traditional sector and the wage in the modern sector. Lewis thought that this might be of the order of 30% - in Kenya, however, at the initial date, the modern sector wage at £ K 124 per year is more than five times the traditional sector wage of £ K 22..

The relationship between the the labour input and output in Kenya - see Panel (10) shows a very flat ( zero or low  $MPL_T$ ) for most of the range, with the impact of labour withdrawals only raising wage rates when more than 80% of the initial traditional labour force has transferred to the modern sector.

The turning point, when the  $MPL_T$  rises above the wage in the traditional sector occurs in 1970 (without any population growth or technical progress it needs to be said) a mere 16 years after the start date. The traditional wage rises fivefold over the subsequent decade, while the modern sector wage rises by about 65% over the same period.

There is a major relocation of the labour force with modern sector labour increasing six-fold, and tradition sector labour falling to a fifth of its original level. The positions of the two sectors in terms of share of total employment, is more-or-less reversed.

Panel (9) shows the rise in the rise in the overall savings rate (savings-to-GDP), which results from increasing share in GDP of the modern sector, which is the only sector that saves (and then only from profits). The slightly surprising feature is that the overall savings rate rises only modestly, form 14% to 18%. The growth rate is impressive, rising from 8% to a peak of 10% at the turning point, when it drops to 6.5% and then slowly declines to around 5% - all this, of course, without any technical progress.

## 6 Using the Model

It is quite instructive to use the model for other developing countries - production functions are not usually available, and in such circumstances the Kenyan coefficients might be used. Data for the initial conditions is generally available (apart from the percentage of surplus labour, which needs an informed guess). The capital stock can be estimated from the rough-and-ready methods outlined in Hodd 1974a.

An Interesting scenario change is to include technical progress in the traditional sector - the estimates in Hodd 1974b indicate a rate of 4.5% a year in Kenya - in other economies this is something that might be influenced by agricultural extension and small-scale credit. The estimated rate of technical progress in the modern sector is around 2% a year - increasing this is probably a more challenging exercise for policy.

The savings rate can perhaps be increased by the actions of the government sector, and several possibilities are suggested by Lewis 1954.

Finally, there is the possibility of reducing the population growth rate. Government policy may be effective (as in China), but a more plausible

refinement for Kenya might be to relate the rate of population growth to the relative sizes of the populations in the traditional sector (predominantly rural) and the modern sector (predominantly urban).

Because of the change in the determination of the model at the turning point, it is not possible with the current construction to insert new parameters, scenario variables, and initial variables and see the results spill out on the 'Display' sheet. For each year after the turning point it is necessary to go to the 'Model' sheet and solve for equilibrium by adjusting modern sector labour. The iterations for equilibrium after the turning point are not arduous, however, and equilibrium can be achieved in under a minute of trial-and-error for each year. It will usually also be necessary to adjust the ranges of the graphs in the 'Display' sheet with each new scenario, so that the relevant sections are shown.

However, the use of versions of this model have been a great success with Development students, even when they have not been specialist in economics. Each student is asked to choose a different developing country and to compare the Lewis model projections with the actual outcomes. It has also been featured in a module for economists covering Economic Modelling.

I am happy to supply copies of the model as an Excel file (requests to [hoddm@wmin.ac.uk](mailto:hoddm@wmin.ac.uk)).

## 7 Further Development

In principle it should be possible to solve for equilibrium after the turning point by using the Solver routine in Excel. Sometimes this works, and sometimes it doesn't - the problem seems to be that the routine will sometimes approach through a route that runs into an illegal operation (division by zero, log of zero, log of a negative number). It should be possible to programme an iterative procedure that avoids these problems - they do not arise in the trial-and-error adjustment of the 'Model' sheet.

More interesting would be a version of the model with the traditional sector identified as agriculture and the modern sector as non-agriculture. This will be a two-good model, and will require a utility function. Such a model might dispense with the section of the traditional production function that has  $MPL_T$  equal to zero, and include the possibility of capital accumulation in traditional agriculture. A two-good, two factor, one consumer general equilibrium model would be required, and it would need to be solved each year as capital accumulates and the work-force increases. The model, with graphical display, in Hodd 2000, drawing on the work of Dinwiddy and Teal 1988, would suffice. As this model determines relative prices, it would shed light on the movement of the terms of trade between the two sectors in a Lewis development process.

Finance through bank credit, a government sector, capital flows and migration, and international trade could all be incorporated as demonstrated, on a piecemeal basis without graphical presentation, in Dinwiddy and Teal (1988).

### Notes

<sup>1</sup>On a personal note, I was a graduate student at Princeton, and I took the Development Economics course taught by Arthur Lewis in the 1965-66 academic year.

Three remarks of the great man remain in the memory. The first was: 'If you are not interested in the plight of small farmers, you are not interested in development.' Initially I thought this was on the grounds that the greatest incidence of poverty was in the rural areas. However, as the course unfolded, it became clear that traditional sector productivity had a key role to play in determining modern sector wages as well.

A second remark was related to the regular overseas missions Arthur Lewis undertook, leading from time-to-time to the rescheduling of the class. After one such mission he confessed that his spirits had sunk when he was joined on the plane by 'an engineer who'd never seen a demand curve, and a sociologist who'd never seen a prosperous capitalist employee'. I leave it to the reader to guess who these two development advisers were.

Finally, Arthur Lewis passed on to the class what he considered to be the most valuable advice he could offer to aspiring international development adviser 'a bottle of champagne and a sleeping tablet - to be taken as soon as you get on the plane'.

At Princeton the original Lewis article was razored out of the 1954 volume of the Manchester School. Interestingly, it has also been razored out of the volume in University College Library. Could this be the work of a demented engineer or an embittered sociologist? More likely it is a sign that the article is heavily consulted. Perhaps an alternative to the citations index could be developed - the number of libraries with the article razored out. Alas, as with all these indicators, it would doubtless lead to perverse behaviour as a junior academic is dispatched to the libraries of the world, just before the Research Assessment Exercise armed with a list of Departmental publications and a cut-throat razor.

## References

- Dinwiddy, C. L. and F. J. Teal. 1988. *The Two Sector General Equilibrium Model: A New Approach*. Oxford: Philip Allan
- Hodd, M. 1974a. 'A Design for an Econometric Model of the Kenyan Economy' *Working Paper No. 171* Nairobi: Institute for Development Studies.
- Hodd, M. 1974b 'Aggregate Supply in Kenya: Production Functions for 4 Major Sectors' *Working Paper No. 173* Nairobi: Institute for Development Studies.
- Hodd, M. 2000. "Walras Without Weeping" Unpublished Public Lecture. Westminster Business School, University of Westminster.
- Lewis, W. A.. 1954. 'Economic Development with Unlimited Supplies of Labour' *Manchester School*.